

A Level Mathematics B (MEI)

H640/03 MEI Pure Mathematics and Comprehension

Question Set 1

- 1 Triangle ABC is shown in Fig. 1.

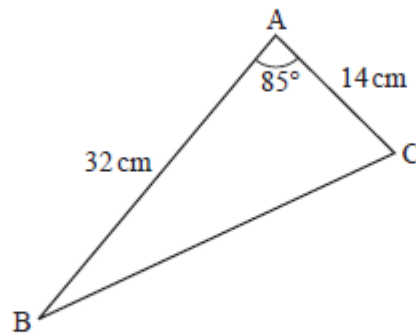


Fig. 1

Find the perimeter of triangle ABC.

[3]

Using the cosine rule: $BC^2 = 32^2 + 14^2 - 2 \times 32 \times 14 \times \cos 85$
 $= 1142$ $BC = 33.8$

$$33.8 + 32 + 14 = 79.8$$

- 2 The curve $y = x^3 - 2x$ is translated by the vector $\begin{pmatrix} 1 \\ -4 \end{pmatrix}$. Write down the equation of the translated curve. [2]

$$y = (x+1)^3 - 2(x+1) - 4$$
$$y = x^3 + 3x^2 + x - 5$$

- 3 Fig. 3 shows a circle with centre O and radius 1 unit. Points A and B lie on the circle with angle $AOB = \theta$ radians. C lies on AO, and BC is perpendicular to AO.

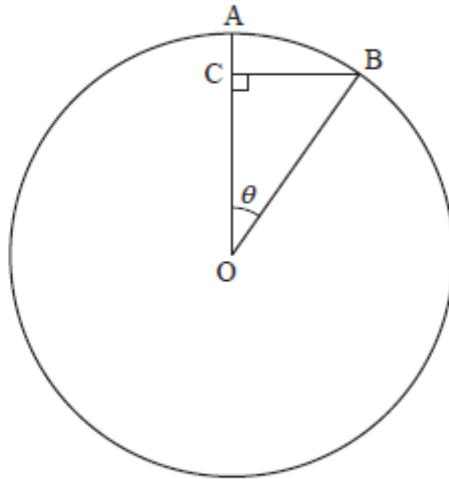


Fig. 3

Show that, when θ is small, $AC \approx \frac{1}{2}\theta^2$.

[2]

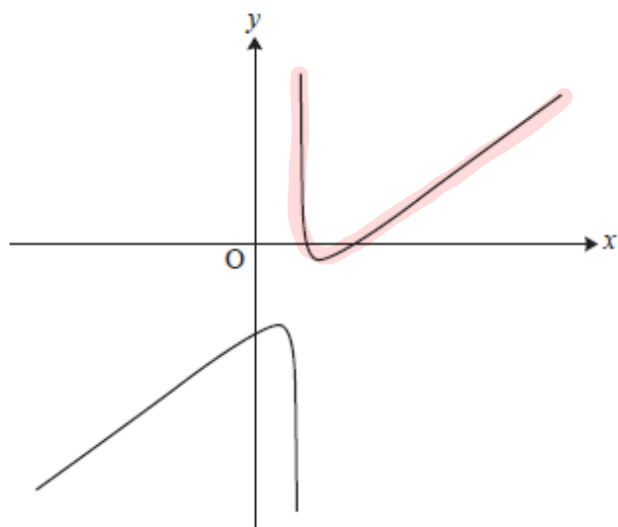
$$OC = 1 \cos \theta = \cos \theta$$

$$\text{When } \theta \text{ is small} = 1 - \frac{\theta^2}{2}$$

$$AC = OA - OC = 1 - \left(1 - \frac{\theta^2}{2}\right) = \frac{\theta^2}{2}$$

4 In this question you must show detailed reasoning.

A curve has equation $y = x - 5 + \frac{1}{x-2}$. The curve is shown in Fig. 4.



(a) Determine the coordinates of the stationary points on the curve.

[5]

$$y = x - 5 + (x - 2)^{-1}$$

$$\frac{dy}{dx} = 1 - (x - 2)^{-2}$$

stationary point $\frac{dy}{dx} = 0$

$$0 = 1 - (x - 2)^{-2}$$

$$\frac{1}{(x - 2)^2} = 1 \rightarrow 1 = (x - 2)^2$$

$$0 = (x - 2)^2 - 1$$

$$0 = x^2 - 4x + 4 - 1$$

$$0 = x^2 - 4x + 3$$

$$0 = (x - 3)(x - 1)$$

$x = 3$ or $x = 1$

$y = 3 - 5 + \frac{1}{3-2}$	$y = 1 - 5 + \frac{1}{1-2}$
$= 3 - 5 + 1$	$= 1 - 5 - 1$
$= -1$	$= -5$

\therefore Stationary points are $(3, -1)$ or $(1, -5)$

(b) Determine the nature of each stationary point.

[3]

from diagram $(3, -1) = \text{minima}$.

$$\frac{d^2y}{dx^2} = 2(x - 2)^{-3} \rightarrow x = 3, 2(3 - 2)^{-3} = 2 \cdot 1^{-3} = \frac{2}{1} = 2$$

$2 > 0 \therefore \text{minima}$

from diagram $(1, -5) = \text{maxima}$

$$\frac{d^2y}{dx^2} = 2(1 - 2)^{-3} = 2(-1)^{-3} = \frac{2}{-1^3} = -2$$

$-2 < 0 \therefore \text{maxima}$

(c) Write down the equation of the vertical asymptote. $x = 2$

[1]

(d) Deduce the set of values of x for which the curve is concave upwards.

[1]

$$x > 2$$

5 A social media website launched on 1 January 2017. The owners of the website report the number of users the site has at the start of each month. They believe that the relationship between the number of users, n , and the number of months after launch, t , can be modelled by $n = a \times 2^{kt}$ where a and k are constants.

(a) Show that, according to the model, the graph of $\log_{10} n$ against t is a straight line. [2]

$$\log_{10} n = \log_{10} (a \times 2^{kt}) \rightarrow \log_{10} (a \times 2^{kt}) = \log_{10} a + kt \log_{10} 2$$

$$\therefore \log_{10} n = \log_{10} a + kt \log_{10} 2$$

= n is a straight line, gradient = $k \log_{10} 2$ which is constant

(b) Fig. 5 shows a plot of the values of t and $\log_{10} n$ for the first seven months. The point at $t = 1$ is for 1 February 2017, and so on.

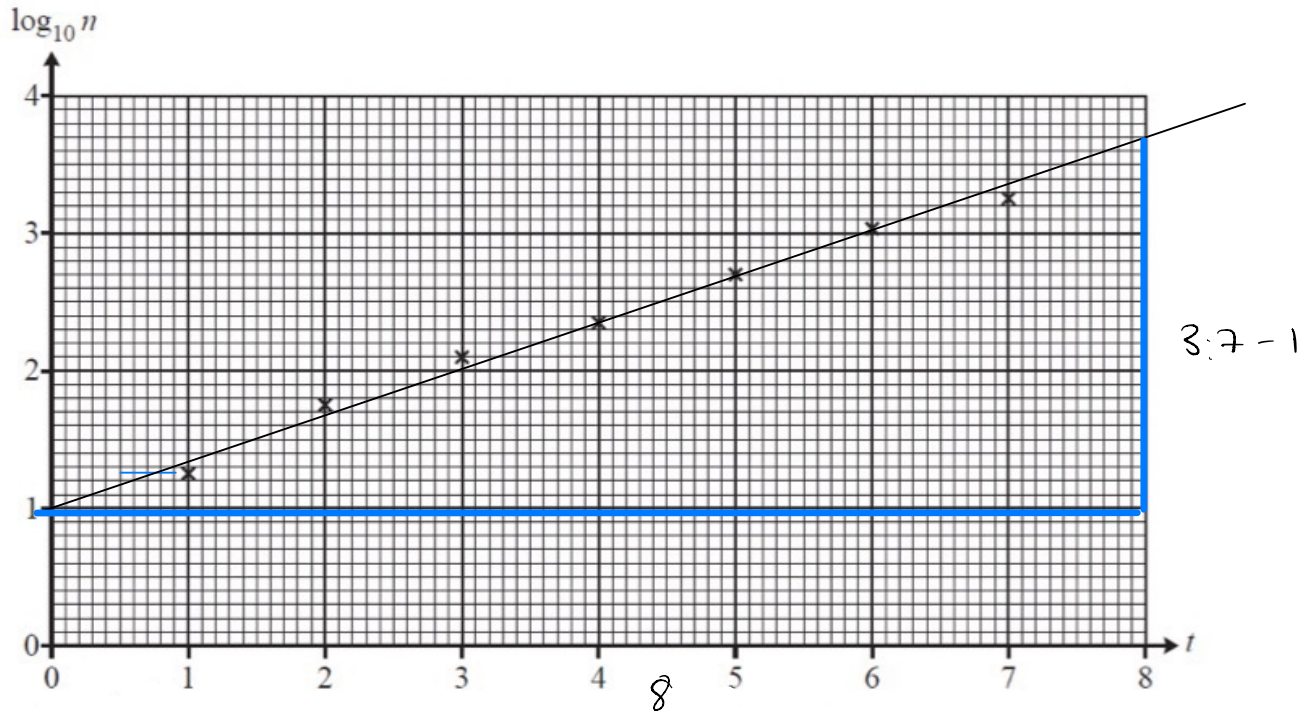


Fig. 5

Find estimates of the values of a and k .

$$\log_{10} n = 1 \quad (\text{y-intercept})$$

$$\therefore \log_{10} a = 1$$

$$a = 10^1 = 10$$

$$\text{gradient} = \frac{3.7 - 1}{8 - 0} = 0.3375$$

$$k \log_{10} 2 = 0.3375$$

$$k = \frac{0.3375}{\log_{10} 2} = 1.12118\dots$$

[4]

$$k = 1.12 \quad (\text{3sf})$$

$$a = 10$$

(c) The owners of the website wanted to know the date on which they would report that the website had half a million users. Use the model to estimate this date. [4]

$$500\,000 = 10 \times 2^{1.12118t}$$

$$1.12118t \log 2 = \log 500\,000$$

$$t = \frac{\log 500\,000}{1.12118 \log 2} = 13.92287409 \approx 14 \text{ months}$$

$$t = 1/3/18$$

(d) Give a reason why the model may not be appropriate for large values of t
The popularity will reach a max

[1]

6

Find the constant term in the expansion of $(x^2 + \frac{1}{x})^{15}$

[2]

$${}^{15}C_5 (x^2)^5 \left(\frac{1}{x}\right)^{10}$$

$${}^{15}C_5 \left(\frac{x^{10}}{x^{10}}\right) = {}^{15}C_5 = 3003$$

7 In this question you must show detailed reasoning.

Fig. 7 shows the curve $y = 5x - x^2$.

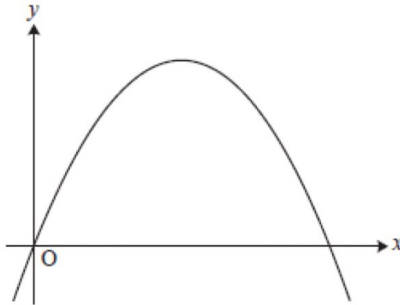


Fig. 7

The line $y = 4 - kx$ crosses the curve $y = 5x - x^2$ on the x-axis and at one other point.

Determine the coordinates of this other point.

[8]

8

(a) In this question you must show detailed reasoning.

$$y = x(5 - x)$$

when $y = 0$, $x = 0$ or $x = 5$

with $y = 4 - kx$, when $x = 0$, $y = 4$ \therefore can not cross there

\therefore must meet at $5, 0$

$$\Rightarrow 0 = 4 - 5k$$

$$5k = 4$$

$$k = 0.8$$

$$\rightarrow 4 - \frac{4}{5}x = 5x - x^2$$

$$20 - 4x = 25x - 5x^2$$

$$5x^2 - 29x + 20 = 0$$

$$x = 5 \quad \text{or} \quad x = \frac{4}{5}$$

$(5, 0) \checkmark$

$$\text{when } x = \frac{4}{5}$$

$$y = 4 - \frac{4}{5} \times \frac{4}{5}$$

$$y = 4 - \frac{16}{25}$$
$$= \frac{84}{25} = 3.36$$

\therefore other point is

$$\left(\frac{4}{5}, \frac{84}{25}\right)$$

8 (a) In this question you must show detailed reasoning.

Determine the gradient of the curve at the point where $t = 1$.

(b) Verify that the cartesian equation of the curve is $x^3 + y^3 = xy$.

no curve given [5]
to answer [3]
∴ can't answer

9 The function $f(x) = \frac{e^x}{1-e^x}$ is defined on the domain $x \in \mathbb{R}, x \neq 0$.

(a) Find $f^{-1}(x)$. [3]

$$\begin{array}{l} y = \frac{e^x}{1-e^x} \\ y - ye^x = e^x \\ y = e^x(1+y) \\ \frac{y}{1+y} = e^x \end{array} \quad \left| \quad \begin{array}{l} \ln \frac{y}{1+y} = x \\ \downarrow \\ \therefore f^{-1}(x) = \ln \frac{x}{1+x} \end{array} \right.$$

(b) Write down the range of $f^{-1}(x)$. $f^{-1}(x) \in \mathbb{R}, f^{-1}(x) \neq 0$ [1]

10 Point A has position vector $\begin{pmatrix} a \\ b \\ 0 \end{pmatrix}$ where a and b can vary, point B has position vector $\begin{pmatrix} 4 \\ 2 \\ 0 \end{pmatrix}$ and point C has position vector $\begin{pmatrix} 2 \\ 4 \\ 2 \end{pmatrix}$. ABC is an isosceles triangle with $AC = AB$.

(a) Show that $a - b + 1 = 0$.

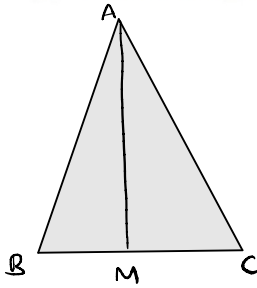
$$AB = \begin{pmatrix} 4 \\ 2 \\ 0 \end{pmatrix} - \begin{pmatrix} a \\ b \\ 0 \end{pmatrix} = \begin{pmatrix} 4-a \\ 2-b \\ 0 \end{pmatrix}$$

$$AC = \begin{pmatrix} 2 \\ 4 \\ 2 \end{pmatrix} - \begin{pmatrix} a \\ b \\ 0 \end{pmatrix} = \begin{pmatrix} 2-a \\ 4-b \\ 2 \end{pmatrix}$$

$$\begin{aligned} \sqrt{(4-a)^2 + (2-b)^2 + 0^2} &= \sqrt{(2-a)^2 + (4-b)^2 + 2^2} \\ (4-a)^2 + (2-b)^2 &= (2-a)^2 + (4-b)^2 + 4 \\ 16 - 8a + a^2 + 4 - 4b + b^2 &= 4 - 4a + a^2 + 16 - 8b + b^2 + 4 \\ 20 - 8a - 4b &= 24 - 4a - 8b \\ 4b &= 4a + 4 \\ 0 &= 4a - 4b + 4 \\ 0 &= a - b + 1 \end{aligned}$$

(b) Determine the position vector of A such that triangle ABC has minimum area.

[6]



M = midpoint of BC

$$M = \begin{pmatrix} 3 \\ 3 \\ 1 \end{pmatrix}$$

$$AM = \begin{pmatrix} 3 \\ 3 \\ 1 \end{pmatrix} - \begin{pmatrix} a \\ b \\ 0 \end{pmatrix} = \begin{pmatrix} 3-a \\ 3-b \\ 1 \end{pmatrix}$$

$$a+1 = b \quad \therefore AM = \begin{pmatrix} 3-a \\ 2-a \\ 1 \end{pmatrix}$$

$$\text{Area} = \frac{1}{2} \times |\vec{AM}| \times |\vec{BC}|$$

$$\begin{aligned} |\vec{AM}| &= \sqrt{(3-a)^2 + (2-a)^2 + 1^2} \\ &= \sqrt{9-6a+a^2 + 4-4a+a^2 + 1} \\ &= \sqrt{14-10a+2a^2} \end{aligned}$$

$$\begin{aligned} \vec{BC} &= \begin{pmatrix} 2 \\ 4 \\ 2 \end{pmatrix} - \begin{pmatrix} 4 \\ 2 \\ 0 \end{pmatrix} = \begin{pmatrix} -2 \\ 2 \\ 2 \end{pmatrix} \quad \therefore |\vec{BC}| = \sqrt{2^2 + 2^2 + 2^2} \\ &= 2\sqrt{3} \end{aligned}$$

$$\begin{aligned} \text{Area} &= \frac{2\sqrt{3} \sqrt{14-10a+2a^2}}{2} = \sqrt{3} \sqrt{14-10a+2a^2} \\ &= \sqrt{3} \sqrt{2(a-2.5)^2 + 0.75} \end{aligned}$$

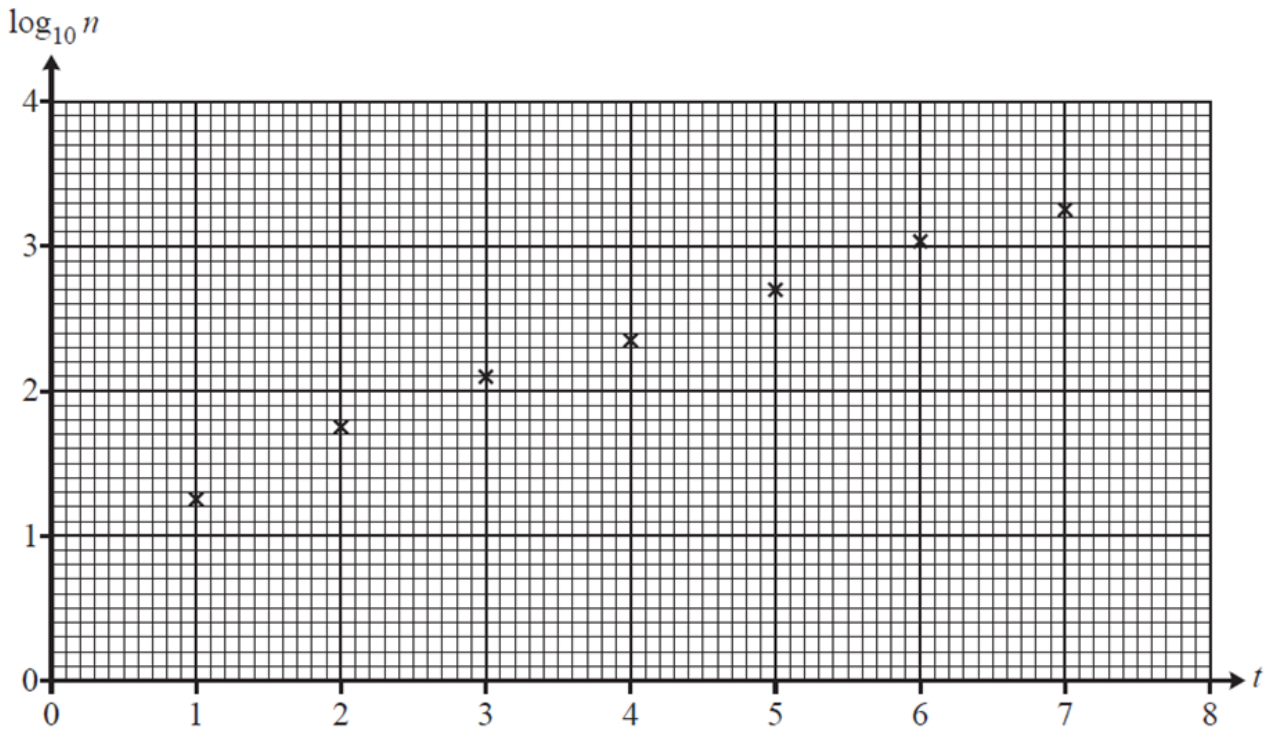
$\therefore a = 2.5$ is the minimum

$$\text{Position vector} = \begin{pmatrix} 2.5 \\ 3.5 \\ 0 \end{pmatrix} \rightarrow b = 2.5 + 1$$

Resource Materials

Question Set No: 1

Fig. 5



OCR

Oxford Cambridge and RSA

Copyright Information

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download from our public website (www.ocr.org.uk) after the live examination series.

If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact The OCR Copyright Team, The Triangle Building, Shaftesbury Road, Cambridge CB2 8EA.

OCR is part of the Cambridge Assessment Group; Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge